

Nonlinear Equation

A non-linear equation can have more than one solution, no real solution, or a solution that turns out to be fake. This section is about picking the right method for the equation in front of you and checking your answer actually works.

1 What counts as a non-linear equation?

A **non-linear equation** contains a variable raised to a power other than one, a variable under a radical, or a variable in a denominator. Anything that keeps the equation from graphing as a straight line. Solving one means finding every value of the variable that makes the equation true.

The SAT tests four types most often: **quadratic**, **radical**, **rational**, and **systems that pair a line with a curve**. Each type has its own solving method, and each has its own way of creating a solution that looks valid but isn't.

Four equation types to recognize.



Quadratic

$$y = ax^2 + bx + c$$

Zero, one, or two real solutions. Solve by factoring, completing the square, or the quadratic formula.



Radical

$$y = \sqrt{x}$$

Isolate the root, then raise both sides to a power. Always recheck solutions in the original equation.



Rational

$$y = 1/x$$

Clear denominators by multiplying through, then exclude any solution that makes a denominator zero



Line + curve system

$$y = ax^2 + bx + c ; y = mx + c$$

Set the expressions equal to each other to find where a line and a parabola meets 0, 1 or 2 points.

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Three ways to solve a quadratic equation.

Factoring -fastest when it works.

Set the equation equal to zero, factor into two binomials, then set each factor equal to zero. Best for equations with integer roots

Quadratic --formula always works

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ plug in **a** **b** and **c** directly from the standard form use this whenever factoring isn't obvious

Square root property-- best for equations with no middle term.

If the equation looks like $(x - h)^2 = k$, take the square root of both sides and remember the plus minus sign.

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Discriminant: How many solutions before you solve?

Discriminant $b^2 - 4ac$	Number of real solutions	What the graph shows
Positive	Two real solutions	Parabola crosses the x-axis twice
Zero	One real solution	Parabola touches the x-axis at its vertex
Negative	No real solutions	Parabola never reaches the x-axis

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Mistakes that quietly cost points



Skipping the check on radical equations. Squaring both sides can introduce a solution that doesn't actually satisfy the original equation. Always substitute back in before finalizing an answer.



Forgetting the $\pm$ when taking a square root $x^2 = 25$ has two solutions, $x = 5$ and $x = -5$, not just one.



Not excluding denominator zero values in rational equations. If clearing denominators produces a solution that makes the original denominator zero, that value must be thrown out.